HEEDS® Technical Tip — Using Constraints to Improve the Resolution of the Pareto Front in a Region of Interest

Level: Beginner

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Introduction

A multi-objective optimization study is typically used to learn about the tradeoffs between two or more objectives. When you perform a multi-objective optimization, HEEDS generates a Pareto front plot like the one shown below (Figure 1).

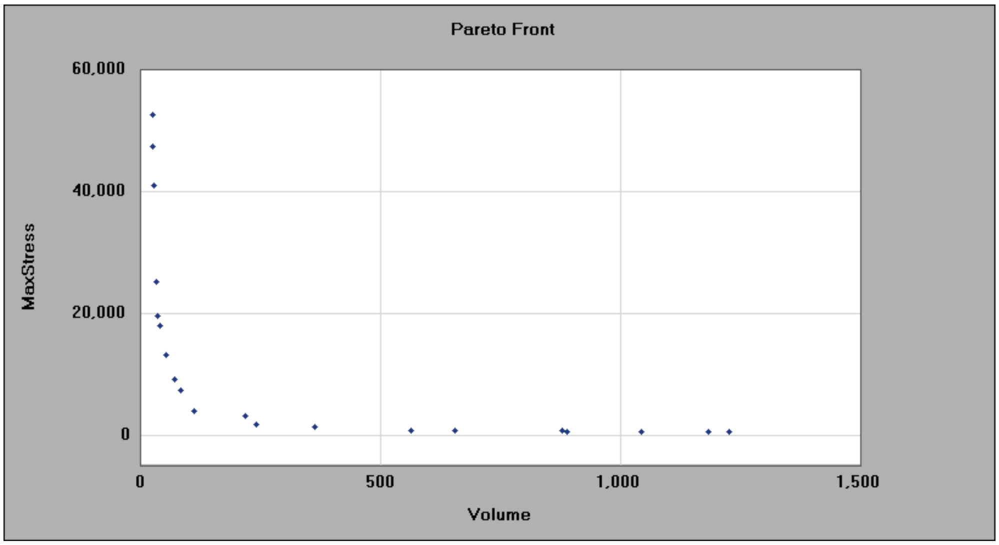


Figure 1. Pareto front plot for a multi-objective optimization study.

Each point on the Pareto front plot represents an optimal design, in the respect that there is no other point in the design space with better values for both of the objectives. The goal of a multi-objective optimization study is to use these design points to identify the region of the Pareto front that represents a good balance between the values of the two objectives. This region is typically referred to as the "knee" of the Pareto front.

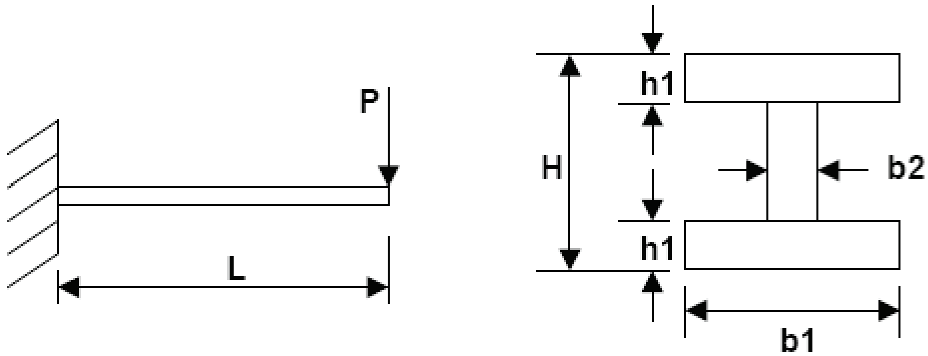
To capture the trade-offs between objectives, it is important to have good resolution in the region of interest on the Pareto front curve. Although the area of interest is typically the "knee" region, in some studies, you may wish to focus on another area of the curve. This is true when you know the range for the objective values within which you wish to understand the trade-offs. In this case, the location of the "knee" area on the global Pareto front is irrelevant (it may or may not lie in the region of interest). Regardless, the resolution of the curve in your area of interest will be improved by increasing the number of points on the Pareto front.

In HEEDS, the resolution of the Pareto front is determined by the *archive size* (the number of points on the plot, per cycle), which you specify. The larger the archive size, the higher the resolution. However, a larger archive size also increases the total number of design evaluations (and, therefore, time) required to generate the optimal Pareto front.

In this tip, we will demonstrate how to effectively improve the resolution of the Pareto front without increasing the archive size.

Example

We will use a simple example to demonstrate the problem and the solution. The optimization statement for the problem is defined below. HEEDS' proprietary MO-SHERPA algorithm is chosen to perform the multi-objective study, with an archive size of 20. The Pareto front from the last cycle is shown in Figure 2.



Minimize:

Volume (Volume of the beam)

MaxStress (Maximum Stress in the beam)

By varying:

0.1 in. ≤ *h1* ≤ 1.0 in.

2.0 in. ≤ *b1* ≤ 12.0 in.

0.1 in. ≤ *b2* ≤ 2.0 in.

3.0 in. ≤ *H* ≤ 7.0 in.

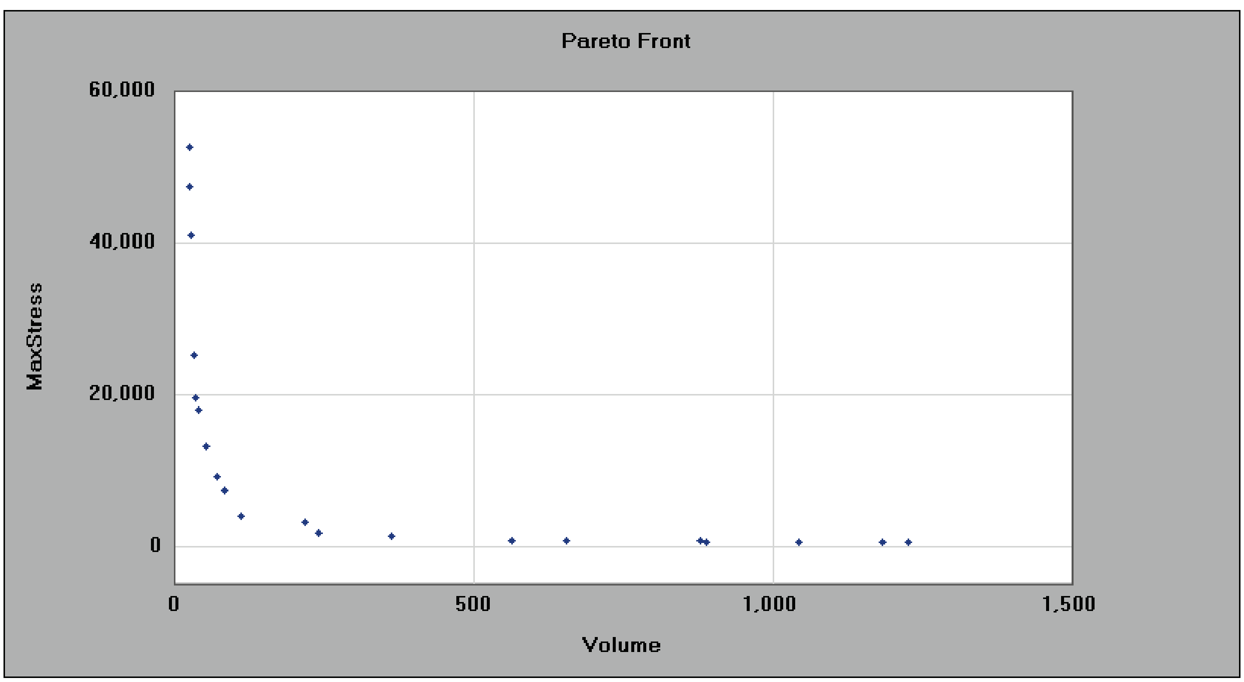


Figure 2. Pareto front plot for the example multi-objective optimization study.

Based on Figure 2, the designs on the Pareto front have values for *Volume* as high as 1,200 and for *MaxStress* as high as 52,000. In many cases, you will not need information about designs at these extreme values and would rather focus on the designs in a region of "reasonable" values for these objectives. For this example, the reasonable values for *MaxStress* and *Volume* are 1000 – 5000 psi and 50 – 250 in^3 respectively. Figure 3 below shows the Pareto front plot for this region of interest.

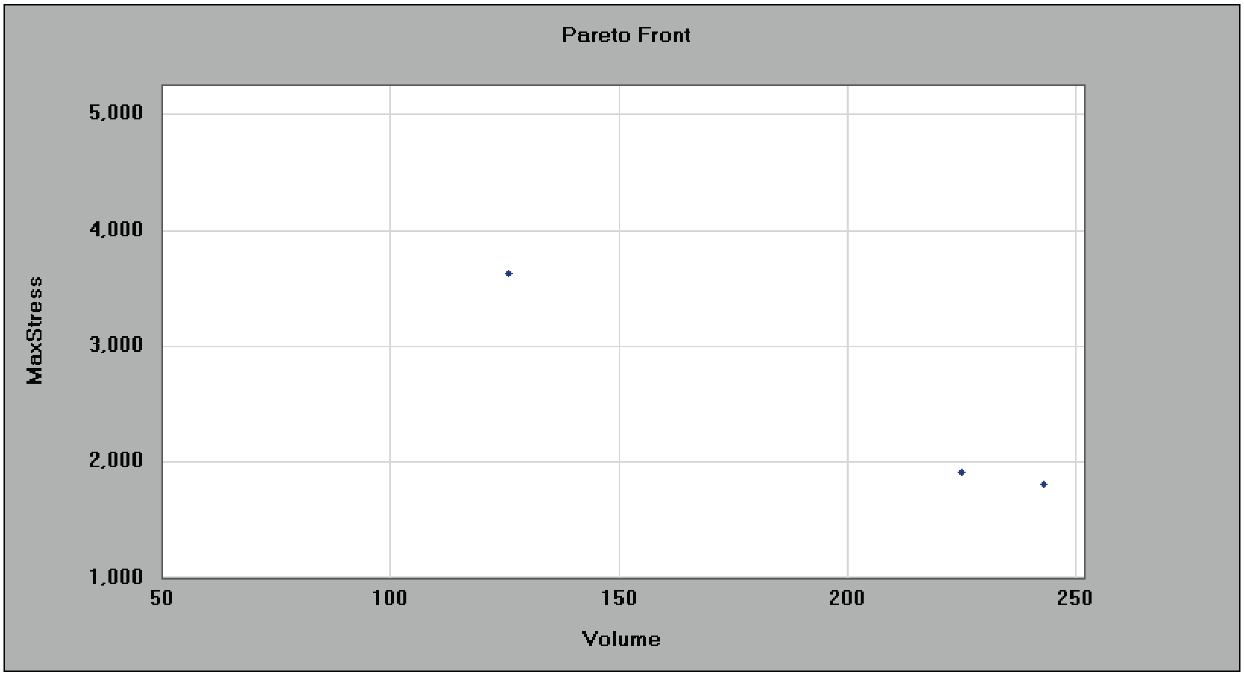


Figure 3. Pareto front plot for the region of interest with new ranges for MaxStress (1000–5000 psi) and Volume (50–250 in³).

By its nature, multi-objective optimization will push the values of the objectives to their extremes in an attempt to capture the entire Pareto front in the design space. Multi-objective optimization will also distribute the designs evenly across the Pareto front to improve the resolution, and this prevents the points from clustering in a specific region. However, if the region of interest is much smaller than the global Pareto front, the trade-off information in this region could be severely limited (as is the case in the example being considered). Notice that there are only three points on the plot in the region of interest (Figure 3).

The Solution

Since you likely want Pareto front information for the objectives within a certain range, you can modify the problem statement to provide better resolution of the Pareto front in that region of interest. This is done by constraining the range of the responses. To do this, you will need to modify the HEEDS setup to include constraints on the two responses (which are also objectives). The modified optimization problem definition is provided below.

Minimize:

Volume (Volume of the beam)

MaxStress (Maximum Stress in the beam)

Subject to:

Volume ≤ 250

MaxStress ≤ 5000

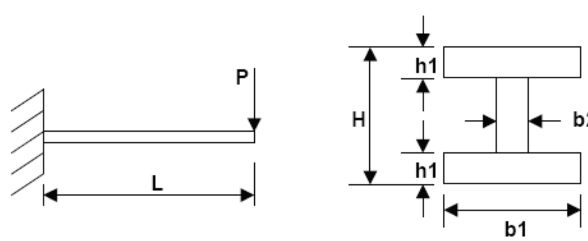
By varying:

0.1 in. ≤ *h1* ≤ 1.0 in.

2.0 in. ≤ *b1* ≤ 12.0 in.

0.1 in. ≤ *b2* ≤ 2.0 in.

3.0 in. ≤ *H* ≤ 7.0 in.



Project Setup and Execution

1. Create two new responses. In this example, we will call them **Volume_Constraint** and **MaxStress_Constraint**.
2. For both of these responses, set the **Source** to **Formula**.
3. In the **Formula Definition** for **Volume_Constraint**, enter **Volume**. For **MaxStress_Constraint**, enter **MaxStress**:

Project Responses

	Name	Source	Formula
1	Volume	File	
▶	MaxStress	File	
3	Volume_Constraint	Formula	Volume
4	MaxStress_Constraint	Formula	MaxStress

4. On the **Assembly** tab, apply the following constraints to the responses:

Agent Responses

	Name	Analysis	Type	Option	Target/Norm
	Volume	Analysis_1	Objective	Minimize	100
	MaxStress	Analysis_1	Objective	Minimize	5000
	Volume_Constraint	Analysis_1	Constraint	<=	250
▶	MaxStress_Constraint	Analysis_1	Constraint	<=	5000

5. Save and run the project.

Results

Although we did not change the archive size, the resulting plot for this example (Figure 5) displays a Pareto front that is much better defined by an increased resolution in the region of interest.

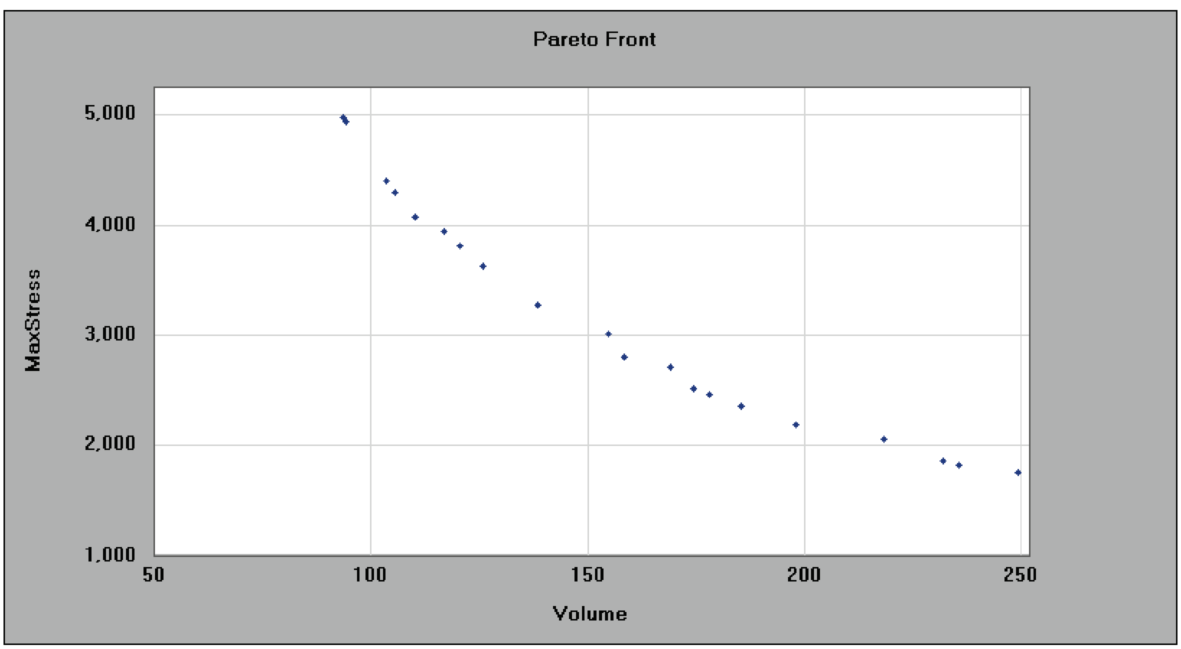


Figure 5. Final Pareto front plot for the multi-objective optimization study.